Quantum Computing
Problem Set 11

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Problem 1: Suzuki-Trotter Formula

The Susuki-Trotter formula reads

$$\exp[(A+B)t] = \lim_{n\to\infty}\left[\exp\left(A\frac{t}{n}\right)\exp\left(B\frac{t}{n}\right)\right]^n \tag{1}$$

Prove equation (1).

Problem 2: Adiabatic Theorem

For a time dependent Hamiltonian $H(t)$, its eigenstates $|\psi_n(t)\rangle$ will also depend on time and the evolution is said to be adiabatic if the system always remains in the same eigenstate. In the lectures we discussed adiabatic quantum computing which relies on the fact that the quantum evolution generated by a time dependent Hamiltonian $H(t)$ can be adiabatic if $H(t)$ changes slowly enough and hence the system always remains in the same eigenstate $|\psi_n(t)\rangle$.

The adiabatic theorem states that a quantum system described by a time dependent Hamiltonian $H(t)$ with non-degenerate eigenstates $|\psi_n(t)\rangle$, that is initially prepared in one of these eigenstates $|\psi_{\overline{n}}(0)\rangle$, will remain in the eigenstate $|\psi_{\overline{n}}(t)\rangle$ provided the change in $H(t)$ is slow enough.

a) For a time dependent $H(t)$, the Schrödinger equation reads

$$\frac{d}{dt}|\psi(t)\rangle = -\frac{i}{\hbar}H(t)\,|\psi(t)\rangle \tag{2}$$

and it's general solution can be written as a linear combination of it's instantaneous eigenstates

$$H(t)\,|\psi_n(t)\rangle = E_n(t)\,|\psi_n(t)\rangle$$

in the form

$$|\psi(t)\rangle = \sum_n c_n(t)\,e^{i\theta_n(t)}\,|\psi_n(t)\rangle \quad \text{where} \quad \theta_n(t) = \frac{-1}{\hbar}\int_0^t ds E(s) \tag{3}$$

How does the expression (3) simplify for a time independent Hamiltonian H? Which quantities become time-independent and which don't?

b) Show that the equation of motion for the time evolution of the expansion factors $c_n(t)$ reads,

$$\dot{c}_n = -c_n\,\langle\psi_n|\dot{\psi}_n\rangle - \sum_{m\neq n}\frac{\langle\psi_n|\dot{H}|\psi_m\rangle}{E_m - E_n}\,e^{i(\theta_m-\theta_n)} \tag{4}$$

where the time dependences of $c_n(t), |\psi_n(t)\rangle$ and $H(t)$ have not been written explicitly.

c) Under which conditions can the second term on the right hand side of equation (4) be neglected? What does this mean for adiabaticity?