Quantum Computing
Problem Set 10

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WiSe 2019-2020

Problem 1: Minimal Toric Code

a) Sketch the qubit grid for the smallest possible realization of the Toric Code.

b) How many stabilizer generators does this implementation have?

c) Describe all possible logical operators.

d) Show that two Pauli operators σ_j^a and σ_l^b (j and l label lattice sites and $a, b = x, y, z$) either commute or anti-commute.

e) What would be the syndrome for a Pauli X error on one of the qubits?

f) Assuming no no errors occur in the detection and correction circuits, which single qubit errors be detected and corrected in this code?

Problem 2: VQE for Heisenberg Dimer

Consider the Hamiltonian

$$H = J(X_1X_2 + Y_1Y_2 + Z_1Z_2) \tag{1}$$

We are interested in the ground state of this Hamiltonian.

a) Form the matrix of the Hamiltonian H and calculate it's ground state and ground state energy.

b) Find an approximation to the ground state of H via the following variational ansatz

$$|\phi(\lambda_b, \lambda_a)\rangle = U_3(\lambda_b)U_2(\lambda_a)U_1 |0,0\rangle \quad \text{where} \tag{2}$$
$$U_3(\lambda_b) = \exp[-i\lambda_b Z_2] \tag{3}$$
$$U_2(\lambda_a) = \exp[-i\lambda_a(X_1X_2 + Y_1Y_2)] \tag{4}$$
$$U_1 = X_1 \tag{5}$$

and $0 \leq \lambda_{a,b} \leq \pi/2$.