Quantum Computing
Problem Set 8

Prof. Dr. Michael J. Hartmann

WiSe 2019-2020

Problem 1: HHL Algorithm

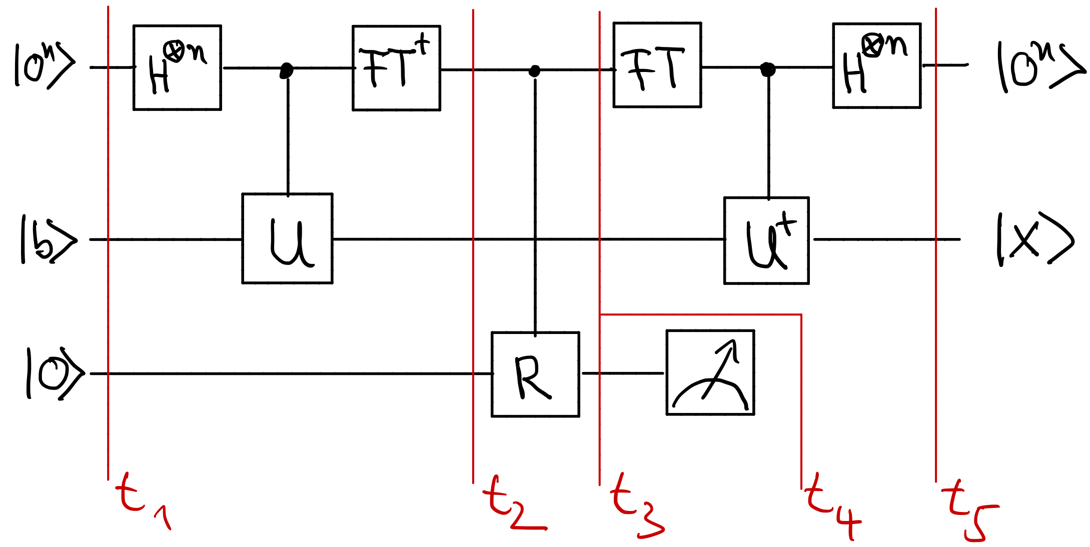
The following circuit executes the HHL algorithm to find $|x\rangle$ such that $A|x\rangle = |b\rangle$, as discussed in the lectures, for

$$|b\rangle = \sum_j \beta_j |u_j\rangle \tag{1}$$

$$U = \exp(2\pi i \kappa A/N) \tag{2}$$

$$A|u_j\rangle = \lambda_J |u_j\rangle \tag{3}$$

where $N = 2^n$, $A^\dagger = A$ and $\lambda_j \leq 1$ for all j.



Only the runs of the circuit where the measurement of the last qubit yields 1 are useful and thus the circuit needs to be repeated until a 1 is measured. The quantity κ is the ration of the ratio of largest to smallest eigenvalue of A (This may be rather large).

a) Compute the state of the qubits at the times t_1, t_2, t_3, t_4 and t_5 indicated in the circuit diagram.

b) How large should one choose the quantum device to be? I.e. how many qubits are needed?

c) How large is the probability that a 1 is measured in the measurement of the bottom qubit? Estimate how often the circuit needs to be run on average (assuming $\kappa \gg 1$).

Problem 2: Application of HHL: polynomial data fitting

Assume we have a set of data points (y_j, t_j), for $j = 1, 2, ..., m$, which we may have obtained from a measurement. For doing further analysis with our data, we now want to fit a polynomial of the form

$$v(t) = x_1 + x_2 t + x_3 t^2 + \cdots + x_n t^{n-1} = \sum_{j=1}^{n} x_j t^{j-1} \tag{4}$$

with $n \ll m$ to it. That is we want to choose the n coefficients x_l $(l = 1, 2, \ldots, n)$ such that the m equations

$$y_j = v(t_j) \tag{5}$$

are fulfilled as good as possible.

a) Re-write equations (5) using the matrix

$$A = \begin{bmatrix} 1 & t_1 & t_1^2 & \ldots & t_1^{n-1} \\ 1 & t_2 & t_2^2 & \ldots & t_2^{n-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & t_m & t_m^2 & \ldots & t_m^{n-1} \end{bmatrix} \tag{6}$$

which is called a "Vandermonde Matrix".

b) For $n \ll m$, the equations (5) cannot be fulfilled exactly. One thus seeks the best approximation to the solution via a linear least squares approximation,

$$\Psi_0 = \min_{\vec{x}} \Psi(\vec{x}) \quad \text{where} \quad \vec{x} = (x_1, x_2, \ldots, x_n)^T \quad \text{and} \tag{7}$$

$$\Psi(\vec{x}) = \sum_{j=1}^{n} |y_j - v(t_j)|^2 \tag{8}$$

Show that $\Psi_0 = \Psi(\vec{x}_0)$, where

$$\vec{x}_0 = (A^T A)^{-1} A^T \vec{y} \tag{9}$$

(you may assume that $A^T A$ is invertible). Hence HHL can be used to fit polynomials to data.