Quantum Computing
Problem Set 6

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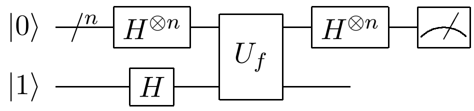
WiSe 2019-2020

Problem 1: Deutsch-Jozsa algorithm

The Deutsch-Jozsa algorithm extends Deutschs algorithm to n qubits. Here, the function $f : \{0,1\}^n \to \{0,1\}$ is either constant, i.e. the output is the same for all possible inputs, or balanced, i.e. the output for exactly half of all inputs is o and for the rest is 1.
The Deutsch-Jozsa algorithm is able to determine whether the function is constant or balanced with one single query of f via a unitary U_f. Note that classically, for the worst case, one would need to test f for half the inputs, an exponentially large number (one can guess the answer with high probability from a much smaller effort, though).
The Deutsch-Jozsa algorithm runs the following circuit on $n+1$ qubits:



(1)

where $-\!\!/^{n}\!-$ is a shorthand for the timeline of n qubits and U_f is as in Deutsch's problem,

$$U_f\,|x_0, x_1, \ldots, x_{n-1}, y\rangle = |x_0, x_1, \ldots, x_{n-1}, y \oplus f(x_0, x_1, \ldots, x_{n-1})\rangle \tag{2}$$

a) For later convenience, first derive a formula for the action of the Hadamard gates on n qubits that are initially in an arbitrary product state.

b) What is the state of the $n+1$ qubits after the first layer of Hadamard gates?

c) What is the state of the $n+1$ qubits after U_f? Write the state in a form such that the last qubit is in the state $(|0\rangle - |1\rangle)/\sqrt{2}$.

d) What is the state of the first n qubits directly prior to the measurement.

e) What does the measurement tell about f being constant or balanced? Hint: Compute the probability to measure all qubits in state $|0\rangle$.

Problem 2: Addition by Fourier transforms

Consider the task of constructing a quantum circuit to compute$|x\rangle \to |x + s \,\text{modulo}\, 2^n\rangle$, where s is a fixed constant and $0 \leq x \leq 2^n$.

a) Show that one efficient way to do this, for values of y such as 1, is to first perform a quantum Fourier transform, then to apply single qubit phase shifts, then an inverse Fourier transform.

b) Draw the full quantum circuit for this computation. You may represent the Fourier transform and its inverse as one gate.

c) How many operations are required and what values of y can be added easily this way?