Quantum Computing
Problem Set 3

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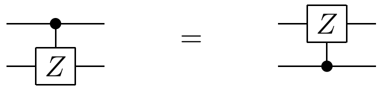
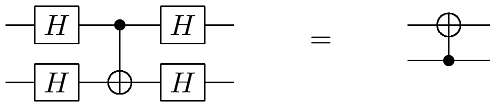
Problem 1: Gate Identities

a) Show that



(1)

b) Show that



(2)

Problem 2: Deutsch algorithm with further ancilla

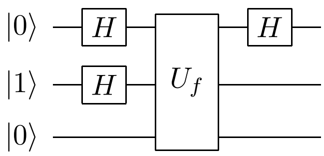
Consider a unitary operator U_f for the Deutsch algorithm, that is constructed with the help of an ancilla qubit that starts in the state $|0\rangle$ and returns to the state $|0\rangle$ at the end of the computation.

$$U_f : \ |j\rangle|k\rangle|0\rangle \ \rightarrow \ |j\rangle \, |k \oplus f(j)\rangle \, |0\rangle, \tag{3}$$

One might think that it doesn't matter whether the ancilla qubit returned to the state $|0\rangle$ since it's state does is not read out anyways. In this problem you should thus investigate what would happen if the ancilla qubit did not return to the state $|0\rangle$, that is if the unitary U_f generated some junk $J(j)$ depending on the input for the first qubit $|j\rangle$ in the ancilla qubit,

$$U_f : \ |j\rangle|k\rangle|0\rangle \ \rightarrow \ |j\rangle \, |k \oplus f(j)\rangle \, |J(j)\rangle. \tag{4}$$

To be specific let's consider the circuit



(5)

a) Would that circuit be able to solve Deutsch's problem if $|J(0)\rangle = |J(1)\rangle$?

b) What would be the measurement outcome for qubit number 1 if $|J(0)\rangle$ and $|J(1)\rangle$ are orthogonal states, $\langle J(0)|J(1)\rangle$?

c) For the case where the ancilla ends up in $|J(j)\rangle$ one could interpret it as some uncontrolled coupling to the environment. Which conclusion can ione thus draw about the effect of such couplings to the environment?