Quantum Computing
Problem Set 1

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Problem 1: Time evolution of a qubit

Just as for vectors one can write quantum states in a specific basis as tuples. One such choice leads to the correspondence

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \leftrightarrow |0\rangle \quad \text{and} \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix} \leftrightarrow |1\rangle \tag{1}$$

for a two-level system (also called a qubit)

a) Write the Hamiltonian

$$H = J\sigma^x \quad \text{where} \quad \sigma^x = |1\rangle\langle 0| + |0\rangle\langle 1| \tag{2}$$

in the representation of (1).

a) For a qubit or two-level system that is in its ground state $|0\rangle$ and whose dynamics is generated by the Hamiltonian H, find the probability to find the qubit in its excited state as a function of time.

Problem 2: Unitary matrices

A unitary matrix U satisfies $U^\dagger U = UU^\dagger = \mathbb{1}$. Show that U is unitary if and only if the inner product of any pair of vectors $\{|a\rangle, |b\rangle\}$ is the same as for the pair $\{U|a\rangle, U|b\rangle\}$

Problem 3: Tensor products

a) For the two states $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$, show that

$$\frac{|+-\rangle - |-+\rangle}{\sqrt{2}} = \frac{|10\rangle - |01\rangle}{\sqrt{2}} \tag{3}$$

b) Two of the Pauli operators are given by

$$\sigma^x = |1\rangle\langle 0| + |0\rangle\langle 1| \quad \text{and} \quad \sigma^y = -i|1\rangle\langle 0| + i|0\rangle\langle 1|. \tag{4}$$

Find the matrix representations of

$$\sigma^x \otimes \sigma^y \quad \text{and} \quad \sigma^y \otimes \sigma^x \tag{5}$$

Note that, for two qubits, the generalization of the dictionary in equation (1) reads,

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leftrightarrow |00\rangle, \quad \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \leftrightarrow |01\rangle, \quad \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \leftrightarrow |10\rangle, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \leftrightarrow |11\rangle. \tag{6}$$

c) Show that

$$(A \otimes B)^\dagger = A^\dagger \otimes B^\dagger \tag{7}$$

$$(A \otimes B)^{-1} = A^{-1} \otimes B^{-1} \tag{8}$$

$$(A \otimes B)(C \otimes D) = AC \otimes BD \tag{9}$$